(10)] with another candidate moment strain

$$\bar{\kappa} = w'' / (1 + w'^2)^{3/2} \tag{11}$$

that, at first, may seem more likely to be correct. The difference in the two geometrically is that while $\bar{\kappa}$ is the curvature of the curve w vs x (which has no physical significance for this problem), κ is approximately (exactly for $s' = 1$) the physical curvature of the beam elastic axis which is nearly inextensional for small strain. Thus, the bending moment is proportional to κ, and the strain energy can now be written as

$$U = \frac{1}{2} \int_0^\ell [EA(s'-1)^2 + EI\kappa^2] \mathrm{d}x \tag{12}$$

The virtual work, including terms associated with a follower force $-Pi(\ell)$, is then given by

$$\delta U + P[\delta\xi(\ell)\cos\beta(\ell) + \delta w(\ell)\sin\beta(\ell)] = 0 \tag{13}$$

with U given by Eq. (12) and s', κ, $\sin\beta$, and $\cos\beta$ expressed from Eqs. (7) and (10). Boundary conditions follow naturally from Eq. (13) that will not and cannot contain quantities like $\sin w'$ as do the boundary conditions depicted in Ref. 1. Thus, it is apparent that the boundary conditions given in Ref. 1 are incorrect, strictly speaking, although numerical results may not be appreciably different when the present equations are used. It is also now apparent that an analysis similar to that of Ref. 1 can be formulated that is restricted to small strains but has no restriction on rotations other than the singularity at $\beta = 90$ deg. A dynamic analysis linearized about the equilibrium condition thus obtained would be consistent with the state of the art in elastic stability theory and may lead to improved understanding of the phenomena discussed in Ref. 1.

References

[1]Kounadis, A. N. and Avraam, T. P., "Linear and Nonlinear Analysis of a Nonconservative Frame of Divergence Instability," *AIAA Journal*, Vol. 19, June 1981, pp. 761-765.

[2]Ziegler, H., *Principles of Structural Stability*, Blaisdell Publishing Company, Waltham, Mass., 1968.

[3]Bolotin, V. V., *Nonconservative Problems of the Theory of Elastic Stability* (English Translation), Pergamon Press, New York, 1963.

[4]Hodges, D. H., Ormiston, R. A., and Peters, D. A., "On the Nonlinear Deformation Geometry of Euler-Bernoulli Beams," NASA TP-1566, April 1980.

[5]Reissner, E., "On One-Dimensional Finite Strain Beam Theory: The Plane Problem," *Journal of Applied Mathematics and Physics* (*ZAMP*), Vol. 23, 1972, pp. 795-804.

AIAA 82-4268

Reply by Authors to D. H. Hodges

A. N. Kounadis* and T. P. Avraam†
National Technical University of Athens
Athens, Greece

THE authors would like to express their thanks to Dr. Hodges for his Comment and to point out the following:

1) An exact investigation of the stability of elastic systems subjected to follower forces entails considerable computational difficulties. This is evident if the mathematical formulation of such a problem is based on small strains but large rotations. The expressions for the axial and moment strain, strain energy, $\sin\beta$, and $\cos\beta$ given by Dr. Hodges are well known.[1-3] The authors, in order to overcome these mathematical difficulties, have chosen to employ the theory of intermediate deformation[4] (small strains but moderately large rotations) which gives excellent results within a wide range.

2) Clearly, the difference between $\sin\beta$ (which theoretically should be present in the boundary conditions) and $\sin w'$ (where $w' = \tan\beta$) due to the small rotations involved is negligible. However, the numerical results presented in the work under discussion are associated with Eqs. (20), which are based on the following aprroximations: $\sin\beta \simeq w'$, $\cos\beta \simeq 1$ and $u = -w''$. These approximations are consistent with the aforementioned theory. A detailed derivation of all nonlinear stability equations of this work is presented in Ref. 5, where use of these approximations is made. The authors are presently employing a more inclusive theory to solve the same problem.

3) An extension of the foregoing work in which several aspects of this problem are clarified is available in Ref. 6.

4) From this work and that of Ref. 6 it is indicated that the nonlinear terms have the following result: the range of values of parameters for which according to linear stability analysis (either dynamic or static) flutter occurs might be appreciably reduced, if a nonlinear static analysis is employed.

Received March 3, 1982; revision received June 7, 1982.

*Associate Professor, Civil Engineering Department.

†Graduate Student, Civil Engineering Department.

References

[1]Novozhilov, V. V., *Foundation of the Nonlinear Theory of Elasticity*, Graylock Press, Rochester, N. Y., 1953.

[2]Pfluger, A., *Stabilitätsproblems der Elastostatik*, Springer-Verlag, Heidelberg, 1950, pp. 9-12.

[3]Dym, C. L. and Shames, I. H., *Solid Mechanics: A Variational Approach*, McGraw-Hill Book Co., New York, 1973.

[4]Brush, P. O. and Almroth, B. O., *Buckling of Bars, Plates and Shells*, McGraw-Hill Book Co., New York, 1975, pp. 3-6.

[5]Avraam, T. P., Pantis, M. A., and Kounadis, A. N., "Snap-Through Buckling of a Simple Frame with a Tangential Load," *Acta Mechanica*, Vol. 36, 1980, pp. 119-127.

[6]Kounadis, A. N., "Postbuckling Analysis of Divergence and Flutter Instability Structures under Tangential Loads," *Proceedings of the 2nd Symposium on Stability in the Mechanics of Continua*, Aug. 31-Sept. 4, 1981, Nümbrecht, W. Germany (to be published).

AIAA 82-4269

Comment on "Potential of Transformation Methods in Optimal Design"

B. Prasad*
Ford Motor Company, Dearborn, Michigan

IN a recent Note,[1] Belegundu and Arora presented an approach for computing the derivatives of a penalty function directly without calculating derivatives of individual constraints. The penalty function acts as an equivalent constraint replacing all other constraints. Their approach permits computation of the first derivatives of the penalty function with one forward and backward substitution (FBS) of the stiffness matrix equation. Newton's method is a powerful technique, for it finds a minimum of a quadratic

Received Nov. 2, 1981; revision received Feb. 4, 1982.

*Engineering and Research Staff.